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CONSTRUCTING OF SURFACES THAT ASSOCIATED TO THE EVOLUTION OF THEIR CURVES

R. A. HUSSIEN & TAHA YOUSSEF

Math Department, Faculty of Science, Assiut Univ., Assiut, Egypt

ABSTRACT

In this paper, we will construct a family of surfaces from a given curve. These surfaces are given as a linear combination of the components of their local coordinate frame. We give some examples about the constructed surfaces such as tubular surfaces and special ruled surfaces (osculating surface, rectifying surface, normal surface and Frenet surface.

KEYWORDS: Curve Evolution; Tubular Surface; Curve Flow; Special Ruled Surface

1. INTRODUCTION

There are a catalogue of surfaces that can be constructed on a curve in the space such as tangent developable surface, normal surface, binormal surface, rectifying developable surface, Darboux developable surface, tangential Darboux developable surface and Hasimoto surfaces which are generated by evolving a regular space curve x by

$$\mathrm{x}_t = \mathrm{x}_s \wedge \mathrm{x}_{ss} = \kappa \mathrm{b}, \tag{1}$$

This is an evolution of the curve in its binormal direction with velocity equal to its curvature and is known as the vortex filament flow. Here, $\mathrm{x}(s,t)$ is a position vector of a point on the curve, t is the time, s is the arc-length parameter, κ is the curvature of x, and b is the unit binormal vector. And the subscripts indicate the differentiation with respect to the indicated variables. The present work continues the program by adding new surfaces to this catalogue. These surfaces are obtained by evolving a regular space curve $\mathrm{r} = \mathrm{r}(u)$ in R^3 as it evolves over time, according to the following evolution equation:

$$\mathrm{x}(u,v) = \mathrm{r}(u) + \alpha(u,v)\mathrm{t} + \beta(u,v)\mathrm{n} + \gamma(u,v)\mathrm{b}, \; 0 \le u \le L, \quad 0 \le v \le V, \tag{2}$$

where $\mathrm{r}(u)$ is the generating curve to the surfaces, u is the arc length parametrization, v represents the time evolution, $(\mathrm{t}, \mathrm{n}, \mathrm{b})$ are the tangent, normal and binormal vectors to the curve. The functions $\alpha(u,v), \beta(u,v)$ and $\gamma(u,v)$ are considered as marching distances of a point unit through the time v in the direction t, n and b respectively, and the position vector $\mathrm{r}(u)$ is seen as the initial location of this point. Here, the values of the functions $\alpha(u,v), \beta(u,v)$ and $\gamma(u,v)$ indicate, respectively, the extension-like, flexion-like, and retortion-like effects, by the point unit through the time v starting from $\mathrm{r}(u)$. Hence, in this paper the functions $\alpha(u,v), \beta(u,v)$ and $\gamma(u,v)$ are denoted as the marching-scale functions in the directions t, n and b, respectively [1].

Evolving of the curves with respect to time has a great interest and has been studied by many authors.

In [2], Hasimoto showed that the motion of an isolated non-stretching thin vortex filament and the family of motions of curves in 3-spaceis described by the nonlinear Schrödinger equation. Lamb [4], used the Hasimoto transformation to connect the motions of curves to the mKdV and sine-Gordon equations.

Nakayama, et al [5], described the motion of curvesand studied the connection between theintegrable evolution equations and the motion of curves in the plane and 3-spaces. Also Nakayama and Wadati [6], studied the motion plane curves in two dimensions. R. Mukherjee and R. Balakrishnan [3],used an approach different from Lamb [4], to map theintegrable nonlinear partial differential equations of real functions to move space curves.

Recently, Nassar, et al [7-9] constructed new geometrical models for motion of plane curves.

Our aim in this paper is to construct a family of surfaces from a given curve. This paper is outlines as follows: In section 2, we introduce some geometric basics on curves and surfaces. In section 3, we study geometric properties of the surfaces generated by Eq. 2 as a special case tubular surfaces and its generator. They are displayed via integration of Serret-Frenet and Gauss-Weingarten equations.

2 THE DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES

2.1 Differential Geometry of Curves

If $\mathrm{x} = \mathrm{x}(s)$ is the position vector of a curve C in space, then the unit tangent t, principal normal n and binormal b vectors vary along C according to the well−known [10]

$$t_s = \kappa n,$$

$$n_s = -\kappa t + \tau b, \tag{3}$$

$$b_s = -\tau n,$$

where s measures arc length along C, κ is its curvature and τ its torsion.

Theorem 2.1 (Fundamental existence and uniqueness theorem for space curves)

Let $\kappa(s)$ and $\tau(s)$ be arbitrary continuous functions on $a \le s \le b$. Then there exists, except for position in space, one and only one space curve C for which $\kappa(s)$ is the curvature, $\tau(s)$ is the torsion and s is a natural parameter along C [14].

2.2 Differential Geometry of Surfaces

We consider a surface imbedded in 3-dimensional Euclidean space R^3. We denote local coordinates of the surface by (u^1, u^2). The surface is specified by the position vector $\mathrm{x}(u^1, u^2)$. We use the Einstein's convention for summation. On the surface there is a metric $g_{\mu\nu}$,

$$g_{\mu\nu} = x_\mu \cdot x_\nu \quad \mu,\nu = 1,2. \tag{4}$$

Here, x_ν is the tangent vector to the surface,

$$\mathrm{x}_{\mu} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{u}^{\mu}}, \quad \mu = 1,2. \tag{5}$$

We denote the inverse of $\boldsymbol{g}_{\mu\nu}$ by $\boldsymbol{g}^{\mu\nu}$. At regular points, where the tangent vectors $\mathrm{t}_1, \mathrm{t}_2$ are linearly independent, we can define the unit normal vector N to the surface,

$$\mathrm{N} = \frac{\mathrm{x}_1 \wedge \mathrm{x}_2}{|\mathrm{x}_1 \wedge \mathrm{x}_2|}. \tag{6}$$

These vectors are related by the Gauss-Weingerten equations [14],

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{u}^{\nu}} \mathrm{x}_{\mu} &= \mathrm{x}_{\lambda} \Gamma^{\lambda}_{\mu\nu} + \mathrm{N} \boldsymbol{L}\mu\nu, \\ \frac{\partial \mathrm{N}}{\partial \boldsymbol{u}^{\nu}} &= -\mathrm{x}_{\lambda} \boldsymbol{g}^{\lambda\mu} \boldsymbol{L}_{\mu\nu}. \end{aligned} \tag{7}$$

In the above, the Christoffel's symbols $\Gamma^{\lambda}_{\mu\nu}$ and the second fundamental form are defined as

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} \boldsymbol{g}^{\lambda\rho} \left(\frac{\partial}{\partial \boldsymbol{u}^{\mu}} \boldsymbol{g}_{\rho\nu} + \frac{\partial}{\partial \boldsymbol{u}^{\nu}} \boldsymbol{g}_{\mu\rho} - \frac{\partial}{\partial \boldsymbol{u}^{\rho}} \boldsymbol{g}_{\mu\nu} \right). \tag{8}$$

$$\boldsymbol{L}_{\mu\nu} = \frac{\partial \mathrm{x}_{\mu}}{\partial \boldsymbol{u}^{\nu}} \cdot \mathrm{N}. \tag{9}$$

From the compatibility conditions of Eq. 7, we get the Gauss and Mainardi-Codazzi equations

$$\begin{aligned} \boldsymbol{L} &= \boldsymbol{g}_{11}((\Gamma^1_{22})_{u^1} - (\Gamma^1_{12})_{u^2} + \Gamma^1_{22}\Gamma^1_{11} + \Gamma^2_{22}\Gamma^1_{12} - \Gamma^1_{12}\Gamma^1_{12} - \Gamma^2_{12}\Gamma^1_{22}) + \boldsymbol{g}_{12}((\Gamma^2_{22})_{u^1} \\ &- (\Gamma^2_{12})_{u^2} + \Gamma^1_{22}\Gamma^2_{11} - \Gamma^1_{12}\Gamma^2_{12}). \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\partial \boldsymbol{L}_{11}}{\partial \boldsymbol{u}^2} - \frac{\partial \boldsymbol{L}_{12}}{\partial \boldsymbol{u}^1} &= \boldsymbol{L}_{11}\Gamma^1_{12} + \boldsymbol{L}_{12}(\Gamma^2_{12} - \Gamma^1_{11}) - \boldsymbol{L}_{22}\Gamma^2_{11}, \\ \frac{\partial \boldsymbol{L}_{12}}{\partial \boldsymbol{u}^2} - \frac{\partial \boldsymbol{L}_{22}}{\partial \boldsymbol{u}^1} &= \boldsymbol{L}_{11}\Gamma^1_{22} + \boldsymbol{L}_{12}(\Gamma^2_{22} - \Gamma^1_{12}) - \boldsymbol{L}_{22}\Gamma^2_{12} \end{aligned} \tag{11}$$

The Gaussian curvature κ_g and the mean curvature κ_m are given by

$$\kappa_g = \boldsymbol{det}(\boldsymbol{g}^{\mu\nu}\boldsymbol{L}_{\nu\lambda}) = \frac{\boldsymbol{L}}{\boldsymbol{g}} = \frac{\boldsymbol{L}_{11}\boldsymbol{L}_{22} - \boldsymbol{L}^2_{12}}{\boldsymbol{g}_{11}\boldsymbol{g}_{22} - \boldsymbol{g}^2_{12}}, \tag{12}$$

$$\kappa_m = \frac{1}{2} \boldsymbol{tr}(\boldsymbol{g}^{\mu\nu}\boldsymbol{L}_{\nu\lambda}) = \frac{\boldsymbol{L}_{11}\boldsymbol{g}_{22} - 2\boldsymbol{L}_{12}\boldsymbol{g}_{12} + \boldsymbol{L}_{22}\boldsymbol{g}_{11}}{2(\boldsymbol{g}_{11}\boldsymbol{g}_{22} - \boldsymbol{g}^2_{12})}. \tag{13}$$

Theorem 2.2 (Fundamental existence and uniqueness theorem of surfaces)

Let $\boldsymbol{g}_{11}, \boldsymbol{g}_{12}$ and $\boldsymbol{g}_{22}$ be functions of s and t of class C^2 and let $\boldsymbol{L}_{11}, \boldsymbol{L}_{12}$ and $\boldsymbol{L}_{22}$ be functions of s and t

of class C^1, all defined on an open set containing (s_0, t_0) such that for all (u^1, u^2):

- $g_{11}g_{22} - g_{12}^2 > 0, \quad g_{11} > 0, \quad g_{22} > 0$
- $g_{11}, g_{12}, g_{22}, L_{11}, L_{12}, L_{22}$ satisfy the compatibility equations (10) and (11).

Then there exists a patch $\mathrm{x} = \mathrm{x}(u^1, u^2)$ of class C^3 defined in a neighborhood of (s_0, t_0) for which $g_{11}, g_{12}, g_{22}, L_{11}, L_{12}, L_{22}$ are the first and second fundamental coefficients. The surface represented by $\mathrm{x} = \mathrm{x}(u^1, u^2)$ is unique except for position in space. [14]

3 Geometric properties of the constructed surfaces based on the Frenet frame of the curve $\mathrm{r} = \mathrm{r}(u)$

In what follows, we establish certain geometric properties of the surfaces based on the Frenet frame of the curve $\mathrm{r} = \mathrm{r}(u)$. Thus we consider a surfaces generated by the vector field $\ell(u, v)$ along the curve. This generator is a linear combination of the Frenet frame ($\mathrm{t}, \mathrm{n}, \mathrm{b}$). Thus we have a surfaces in the form

$$\mathrm{x}(u, v) = \mathrm{r}(u) + \ell(u, v). \tag{14}$$

Explicitly, we obtain the motion of a point on the surface that specified by

$$\mathrm{x}(u, v) = \mathrm{r}(u) + \alpha(u, v)\mathrm{t} + \beta(u, v)\mathrm{n} + \gamma(u, v)\mathrm{b}. \tag{15}$$

The tangent space to S at an arbitrary point $P = \mathrm{x}(u, v)$ of S is spanned by

$$\begin{aligned} \mathrm{x}_u &= (1 + \alpha_u - \beta\kappa)\mathrm{t} + (\kappa\alpha + \beta_u - \tau\gamma)\mathrm{n} + (\tau\beta + \gamma_u)\mathrm{b} \\ \mathrm{x}_v &= \alpha_v\mathrm{t} + \beta_v\mathrm{n} + \gamma_v\mathrm{b} \end{aligned} \tag{16}$$

Furthermore, the coefficients of the first fundamental form are

$$\boxed{\begin{aligned} g_{11} &= (1 + \alpha_u - \beta\kappa)^2 + (\kappa\alpha + \beta_u - \tau\gamma)^2 + (\tau\beta + \gamma_u)^2 \;, \\ g_{12} &= \alpha_v(1 + \alpha_u - \beta\kappa) + \beta_v(\kappa\alpha + \beta_u - \tau\gamma) + \gamma_v(\tau\beta + \gamma_u)\;, \\ g_{22} &= \alpha_v^2 + \beta_v^2 + \gamma_v^2 \;, \end{aligned}} \tag{17}$$

so the fundamental metric is

$$\begin{aligned} g &= [(1 + \alpha_u - \beta\kappa)^2 + (\kappa\alpha + \beta_u - \tau\gamma)^2 + (\tau\beta + \gamma_u)^2][\alpha_v^2 + \beta_v^2 + \gamma_v^2] \\ &\quad - [\alpha_v(1 + \alpha_u - \beta\kappa) + \beta_v(\kappa\alpha + \beta_u - \tau\gamma) + \gamma_v(\tau\beta + \gamma_u)]^2 . \end{aligned} \tag{18}$$

The unit normal normal vector field along the surface S is given as

$$\begin{aligned} \mathrm{N} &= \frac{1}{\sqrt{g}}[[\beta\tau\beta_v - \gamma_v(\alpha\kappa - \gamma\tau + \beta_u) + \beta_v\gamma_u]\, t + [-\beta(\tau\alpha + \kappa\gamma_v) + \gamma_v(1 + \alpha_u) - \alpha_v\gamma_u]\mathrm{n} \\ &\quad + [\alpha\kappa\alpha_v + \beta\_v(-1 + \beta - \alpha_u) + \alpha_v(-\gamma\tau - \beta_u)]\mathrm{b}] \end{aligned} \tag{19}$$

Differentiating x_s, x_t with respect to s and t yields

$$\begin{aligned}
\mathrm{x}_{uu} &= [-\beta\kappa' - \kappa\beta_u - \kappa(\alpha\kappa - \gamma\tau + \beta_u + \alpha_{uu})]\mathrm{t} \\
&+ [\alpha\kappa' - \gamma\tau' + \kappa\alpha_u + \kappa(1 - \beta\kappa + \alpha_u) - \tau\gamma - \tau(\beta\tau + \gamma_u + \beta_{uu})]\mathrm{n} \\
&+ [\beta\tau' + \tau\beta_u + \tau(\alpha\kappa - \gamma\tau + \beta_u) + \gamma_{uu}]\mathrm{b}\ , \\
\mathrm{x}_{uv} &= [-\kappa\beta_v + \alpha_{uv}]\mathrm{t} + [\kappa\alpha_v - \tau\gamma_v + \beta_{uv}]\mathrm{n} + [\tau\beta_v + \gamma_{uv}]\mathrm{b}\ , \\
\mathrm{x}_{vv} &= \alpha_{vv}\mathrm{t} + \beta_{vv}\mathrm{n} + \gamma_{vv}\mathrm{b}\ .
\end{aligned} \tag{20}$$

A short calculation shows that the coefficients of the second fundamental form are

$$\begin{aligned}
L_{11} &= \frac{1}{\sqrt{g}}[[\beta\tau\beta_v - \gamma_v(\alpha\kappa - \gamma\tau + \beta_u) + \beta_v\gamma_u][-\beta\kappa' - \kappa\beta_u - \kappa(\alpha\kappa - \gamma\tau + \beta_u + \alpha_{uu})] \\
&+ [-\beta(\tau\alpha + \kappa\gamma_v) + \gamma_v(1 + \alpha_u) - \alpha_v\gamma_u] \\
&\times[\alpha\kappa' - \gamma\tau' + \kappa\alpha_u + \kappa(1 - \beta\kappa + \alpha_u) - \tau\gamma - \tau(\beta\tau + \gamma_u + \beta_{uu})] \\
&+ [\alpha\kappa\alpha_v + \beta_v(-1 + \beta - \alpha_u) + \alpha_v(-\gamma\tau - \beta_u)][\beta\tau' + \tau\beta_u + \tau(\alpha\kappa - \gamma\tau + \beta_u) + \gamma_{uu}]]\ , \\
L_{12} &= \frac{1}{\sqrt{g}}[[\beta\tau\beta_v - \gamma_v(\alpha\kappa - \gamma\tau + \beta_u) + \beta_v\gamma_u][-\kappa\beta_v + \alpha_{uv}] \\
&+ [-\beta(\tau\alpha + \kappa\gamma_v) + \gamma_v(1 + \alpha_u) - \alpha_v\gamma_u][\kappa\alpha_v - \tau\gamma_v + \beta_{uv}] \\
&+ [\alpha\kappa\alpha_v + \beta_v(-1 + \beta - \alpha_u) + \alpha_v(-\gamma\tau - \beta_u)][\tau\beta_v + \gamma_{uv}]]\ , \\
L_{22} &= \frac{1}{\sqrt{g}}[[\beta\tau\beta_v - \gamma_v(\alpha\kappa - \gamma\tau + \beta_u) + \beta_v\gamma_u]\alpha_{vv} + [-\beta(\tau\alpha + \kappa\gamma_v) + \gamma_v(1 + \alpha_u) - \alpha_v\gamma_u]\beta_{vv} \\
&+ [\alpha\kappa\alpha_v + \beta_v(-1 + \beta - \alpha_u) + \alpha_v(-\gamma\tau - \beta_u)]\gamma_{vv}]\ .
\end{aligned} \tag{21}$$

The set of equations (17, 21) is the main result of this paper. For a given marching-scale functions $\alpha(u,v), \beta(u,v)$ and $\gamma(u,v)$, the surface is determined from these equations. In the next subsections, we shall study some special surfaces.

3.1 TUBULAR SURFACES

In this subsection we study tubular surfaces which can be obtained via evolving a regular space curve in space.

The tubular surface associated to the space curve r is a surface swept by a family of spheres of constant radius (radius of the tube), having the center on the given curve.

If we choose marching-scale functions such that $\alpha = 0, \beta = \rho \sin v, \gamma = \rho\cos v$. Then from eq.(15), we have the well known tubular surface of radius $\rho > 0$ around the curve r, which has the following representation [10], [11]

$$\mathrm{x}(u,v) = \mathrm{r}(u) + \rho(\sin v\mathrm{n} + \cos v\mathrm{b}), \tag{22}$$

where $a \le u \le b$, $\mathrm{n}(u), \mathrm{b}(u)$ are the principal normal and binormal vectors of r, respectively.

The coefficients of the first and second fundamental forms for this surface are given from Eqs. (17) and (21). In case of $\alpha = 0, \beta = \rho\sin v, \gamma = \rho\cos v$, we have:

$$\begin{aligned} g_{11} &= (-1+\rho\kappa\cos v)^2 + \rho^2\tau^2, \\ g_{12} &= \rho^2\tau, \\ g_{22} &= \rho^2, \\ L_{11} &= \kappa(-1+\rho\kappa\cos v)\cos v + \rho\tau^2, \\ L_{12} &= \rho\tau, \\ L_{22} &= \rho. \end{aligned} \tag{23}$$

Thus, the Gaussian and mean curvature for the tube surfaces $x(u,v)$ are computed as

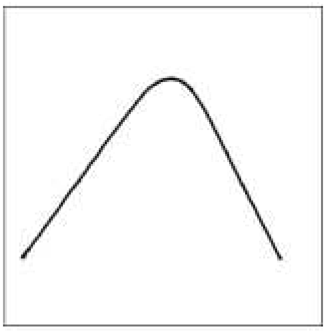
$$\kappa_g = \frac{-\kappa\cos v}{\rho(1-\rho\kappa\cos v)}, \quad \kappa_m = \frac{1}{2}\left(\frac{1}{\rho} + \kappa\rho\right). \tag{24}$$

3.2 Geometric Visualization Tubular Surfaces and Its Generator

In this subsection, we display some curves and its tubular surfaces. The problem of constructing curves through κ, τ analytically is very difficult. Thus we try to obtain the space curves numerically. In [15], one can plot the curve numerically.

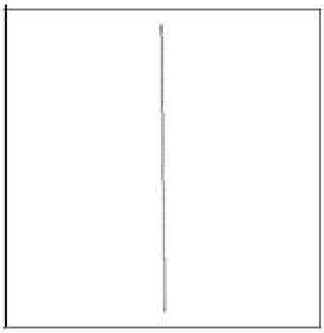
Similarly, the problem of constructing surfaces through g_{ij}, L_{ij} analytically is very difficult. Thus we try to obtain the surfaces numerically. In [16], one can check the integrability conditions and plot the surface numerically.

Now for a given κ, τ, the program [15] produce the curves below, the set of equations (23) and the program [16] produce the surfaces below.

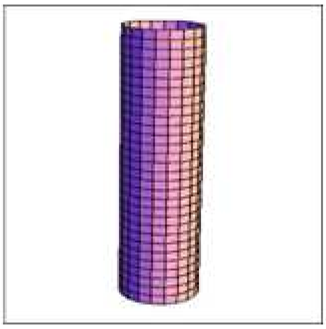


Plane curve with $(\kappa = sech^2(u),\ \tau = 0)$

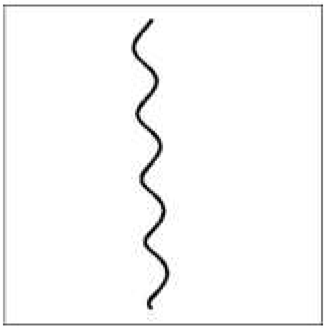
Tube surface around the juxtaposed curve



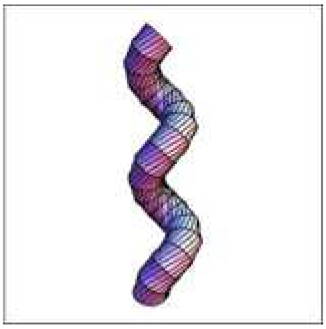
Plane curve with $(\kappa = 0, \tau = 0)$



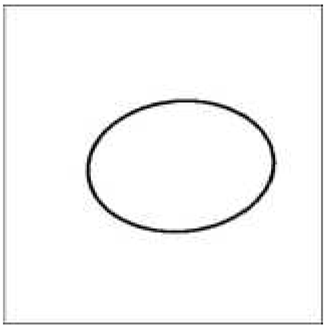
Tube surface around the juxtaposed curve



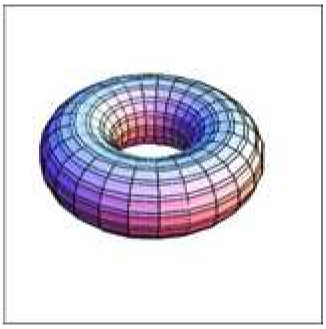
Space curve with $(\kappa = 1, \tau = 1)$



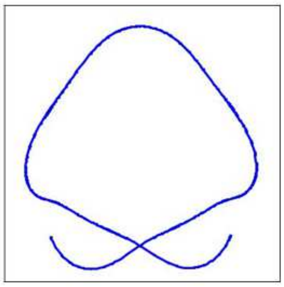
Tube surface around the juxtaposed curve



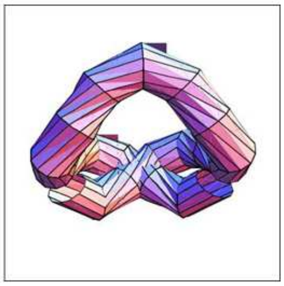
Plane curve with $(\kappa = 1, \tau = 0)$



Tube surface around the juxtaposed curve



Space curve with $(\kappa = \cos u, \tau = \sin u)$



Tube surface around the juxtaposed curve

Space curve with $(\kappa = 1, \tau = u)$

Tube surface around the juxtaposed curve

4. SPECIAL RULED SURFACE

A ruled surface is a surface generated by a straight line moving along a curve.

Definition 4.1. A ruled surface M in R^3 is a surface which contains at least one 1-parameter family of straight lines.

Thus a ruled surface has a parametrization $x : U \to M$ of the form

$$\mathrm{x}(u,v) = \mathrm{r}(u) + v\ell(u), \tag{25}$$

where $\mathrm{r}(u)$ is called the directrix of the surface M (also called the base curve) and $\ell(u)$ is the director curve. The straight lines themselves are called rulings. The rulings of a ruled surface are asymptotic curves. Furthermore, the Gaussian curvature on a ruled regular surface is everywhere nonpositive [10].

In this subsection, we study spacial ruled surface generated by linear combination of $\{\mathrm{t},\mathrm{n}\}$, $\{\mathrm{t},\mathrm{b}\}$, $\{\mathrm{n},\mathrm{b}\}$, $\{\mathrm{t},\mathrm{n},\mathrm{b}\}$, we called these surfaces osculating surface, rectifying surface, normal surface and Frenet surface associated to space curve, respectively.

We construct ruled surfaces depending on the marching-scale functions (α,β,γ) as the following

1. $\alpha = \gamma = v, \beta = 0$
2. $\alpha = \beta = v, \gamma = 0$
3. $\beta = \gamma = v, \alpha = 0$
4. $\beta = \gamma = \alpha = v$

According to these conditions, we obtain four representations for ruled surfaces as follows

$$\mathrm{x}(u,v) = \mathrm{r}(u) + v(\mathrm{t}(u) + \mathrm{n}(u)). \tag{26}$$

$$\mathrm{x}(u,v) = \mathrm{r}(u) + v(\mathrm{t}(u) + \mathrm{b}(u)). \tag{27}$$

$$\mathrm{x}(u,v) = \mathrm{r}(u) + v(\mathrm{n}(u) + \mathrm{b}(u)). \tag{28}$$

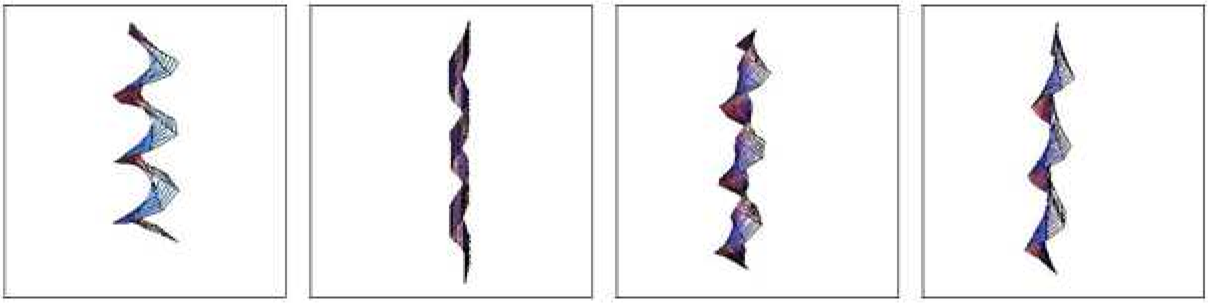
$$\mathrm{x}(u,v) = \mathrm{r}(u) + v(\mathrm{t}(u) + \mathrm{t}(u) + \mathrm{b}(u)). \tag{29}$$

These surfaces are called osculating surface, rectifying surface, normal surface and Frenet surface, associated to space curve respectively. The fundamental quantities g_{ij}, L_{ij} can be obtained from equations (17) and (21).

4.1 Geometric Visualization of Spacial Ruled Surfaces Associated to Space Curve

In this subsection, we shall display special ruled surfaces from its fundamental quantities g_{ij}, L_{ij} by using strategy as in subsection 3.2.

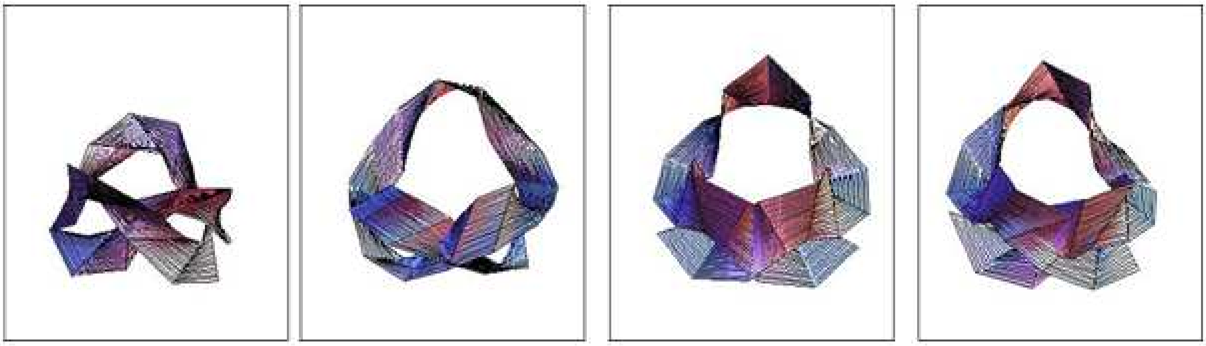
(i)MODEL 1



(a) Osculating Surface (b) Rectifying Surface (c) Normal Surface (d) Frenet Frame field Surface

Figure 1: Ruled Surfaces Attached to the Curve $\kappa = 1$, $\tau = 1$

(ii)MODEL 2



(a) Osculating Surface (b) Rectifying Surface (c) Normal Surface (d) Frenet Frame Field Surface

Figure 2: Ruled Surfaces Attached to the Curve $\kappa = \cos u$, $\tau = \sin u$

CONCLUSIONS

We constructed a family of surfaces from a given curve. The surface is given as a linear combination of the components of its local coordinate frame. The coefficients of the fundamental forms are derived. Tubular surfaces and special ruled surfaces(osculating surface, rectifying surface, normal surface and Frenet surface) are plotted via numerical integration of Gauss-Weingarten equations.

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